Shape and structure in conceptual design of bridges

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INTRODUCTION

Models have always been useful instruments to foresee the characteristics of a work. Aristotele tells about models suggested by architects of that period, in competition one against the other, who showed their work to the client to make him appreciate aesthetical and functional characteristics. Only after Galilei this custom entered the heritage of Science and then models became also tools to simulate and to analyse structural behaviour; in the nineteenth century Sir Benjamin Baker made a “human cantilever” model [1] of his Forth Railway Bridge with his Japanese assistant Mr. Wantanabe representing the live load in the middle (Fig.1).

Fig. 1. The “human model” of the Forth Railway Bridge made by Sir Benjamin Baker.

Nowadays, a model has to show how shape, structure and environment become integrated in perfect harmony. A model should not be only a method to understand structural behaviour, but also a clear and effective instrument of investigation and judgement. At present, separation between architects and engineers is becoming wider and wider especially in bridges design [1]. Cooperation between architects and engineers is a question of individuals and the chemistry between them as Sir Norman Foster says [2]; but not only. Pablo Picasso used to say [3] that the best bridge is the one which could be reduced to a thread, a line, without anything left over; which fulfilled strictly its function of uniting two separated distances. Bridges should be light, transparent. Their beauty, in general, should come from their economic structural shape: in geometrical contours of bridges, maybe more than in other architectural works, it is possible to recognise the expression, almost exclusive, of structural functionality. More than in other civil engineering works, load
paths in bridges draw their profile. That is why, at present, the right model in bridge design is the one that opens new prospects in the search for a common language between engineers and architects to give voice, in harmony and in a single design, to formal, aesthetical, functional and structural aspects.

**Keywords:** bridge design, Strut and Tie Model, Load Path Method, strain energy

**LOAD PATH METHOD: BASIC PRINCIPLES**

Born as a method to design Strut and Tie Models [4] in reinforced concrete structures (Art. 5.6.4 EN 1992-1-1 [5]), the Load Path Method (whose basic principles are more widely illustrated in references [6-13]) is a clear and effective technical instrument of investigation and judgement. Not only a numerical but also a geometrical method that predicts calculation results disclosing the shape aspects from which it is possible to recognise real structural behaviour.

The most suitable orthogonal Cartesian system of architectural forces to physical environment in which they flow is the one capable to bring back them only to vertical loads and horizontal thrusts. According to this, structure can be read as the trace of loads path [10]. The form of the structure is the result of their mutual integration and mainly of the influence of profiles traced by thrusts path, forced to deviate their natural horizontal flows to the soil, in order to go in search of equilibrium. Forces represent loads that, in the way from their application points (S) to the restraints (E), in every deviation node, have to apply thrusts (H) to the rest of the structure and to receive deviation forces equal in value and opposite in direction to thrusts in order to respect equilibrium (Fig. 2). The load path represents the line along which a force or a force component (more precisely: the component of a force in a chosen direction, e.g. the vertical component of a load) is carried through a structure from the point of loading to its support [18]. The force component (F in Fig. 2) associated with a load path remains constant on its way through the structure; as a consequence of this definition, thrust H must be perpendicular to the travelling load F.

![Fig. 2. Load Path (LP) and Strut and Tie Model (STM).](image)

The design of this load flowing through the structure can be approximated by polygonal lines in which there are thrusts in every deviation node. Structure will be crossed by fluxes in compression (dashed lines), when loads travel in the same direction of their path, and by fluxes in tension (continuous lines) along which loads go in the opposite direction respect to their path. According to classical theory, the basic principles that lead Load Path Method (LPM) are the respect of equilibrium and consistency. Thrusts in deviation nodes are necessary in order to respect equilibrium and every path is possible if it is equilibrated. Among infinite paths in equilibrium, loads have to choose the one in which their vectors invest the minimum quantity of strain energy, that is the only one equilibrated and consistent. At this purpose loads get energy from their own potential energy that decreases. Along a generic path (polygonal in this model), the calculus of the invested strain energy (D) is simplified in the summation of the terms relative to each side of the traverse:
\[ D = \frac{1}{2} \sum N_i l_i \varepsilon_i \]  

where “i” is the generic side of the load path, \( N_i \) is the intensity of the vector that brings load on that side of the load path, \( l_i \) is the length of the generic side and \( \varepsilon_i \) is the relative strain that is medium constant on \( l_i \).

LOADS PATH IN CABLE SUPPORTED BRIDGES: SUSPENSION AND STAYED

For long spans, cable supported bridges are by far the most adaptable because of the following [14]:

- vertical loads move from the deck to the pylons using cable working only in tension, their ideal way to resist;
- the cable is formed by thin elements which, apart from providing its flexibility, enable its resistance capacity to be increased to a maximum;
- if formed by thin wires or strands, the cable can be formed wire by wire, enabling large diameter cables to be made in large span bridges with small-sized erecting equipment.

The consequence on the environment is that cable supported bridges are very light and transparent because pylons are the only heavy parts.

In the following, the interpretation of cable supported bridges behaviour by Load Path Method is presented; the method immediately exhibits the correlation between form (geometry) and structure (distribution of loads and of thrusts): a particularly narrow correlation in the case of the cable supported bridges. The case of bridges geometrically symmetrical and symmetrically loaded only with vertical loads will be considered. Despite this assumption, that benefits the simplicity of the analysis, the method has general validity.

Loads path in suspension bridges

Like the arch [12], the suspension bridge is a structure which resists thanks to its shape. Vertical loads move from their application points to the bottom of the suspenders thanks to the deck and then, inside the suspenders, by a path in tension, they move from the deck to the main cable (Fig. 3).

![Fig. 3. Loads path in a suspension bridge.](image)

The main cable behaves like an inverse arch. This correlation between form and static is well shown in Fig. 4: it is sufficient to observe the correspondence between the cable geometry, the load path (Fig. 4a) and the geometry of the loads equilibrium polygon (Fig. 4b). The loads equilibrium polygon also shows that if the direction of every load is from the top to the bottom, every inclination angle \( \theta_i \) is not bigger than \( \theta_1 \). According to LPM, the reason of this is that, going from the pylon to the centre of the bay, the travelling load, decreasing itself, has to reduce the inclination of its path to
keep constant the value of the thrust \( H \) (Fig. 4b). In the assumption that the direction of every load is from the top to the bottom, in every node \( O_i \) the inclination angle of the \( \sum F_j \) path must increase \((\theta_i > \theta_{i+1})\) because in every \( O_i \), \( F_i \) deviates and applies to the structure a thrust:

\[
H_i(F_i) = F_i \cot \theta_i
\]

For equilibrium reasons (= to make possible the \( F_i \) deviation) \( \sum F_j \), coming from the top, must deviate in \( O_i \) of \( \delta_i \), increasing the inclination angle from \( \theta_{i+1} \) to \( \theta_i \), and must apply a thrust equal in value (but opposite in direction) to the \( F_i \) one (Fig. 4c):

\[
H_i(\sum F_j) = \sum F_j (\cot \theta_{i+1} - \cot \theta_i)
\]  
(2)

Because of the equilibrium of these two thrusts it is possible that loads go from their application points to the pylons only by a path in tension.

Equation (2) analytically shows what Fig. 4 exhibits graphically: the line of the “possible” (= in equilibrium) LP is strictly related to the intensity and distribution of loads \( F_i \).

It is also possible to interpret the loads \( \sum F_j \) path as the one of the thrust \( H \) (Fig. 5). In this case, the travelling load does not change (if every \( F_i \) is vertical). In every node the thrust path (from the pylon to the centre) deviates of \( \delta_i \), changing its inclination from \( \theta_i \) to \( \theta_{i+1} \). This is possible (= in equilibrium) because in every node \( O_i \), the vertical thrust \( V_i \) (directed from the bottom to the top) of \( H \) is equilibrated by the load \( F_i \) which is equal in value but opposite in direction.

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**Fig. 4.** The main cable as path of vertical loads.

**Fig. 5.** The main cable as path of the thrust.

Even if, as above mentioned, there is a strict similarity between the main cable and a arch, there is also a
basic difference. In fact, the suspension bridge main cable because of its lack of bending stiffness, must always coincide with the load system antifunicular polygon; this is why, when the system changes, the cable must change shape to fit the funicular again.

**Loads path in cable-stayed bridges**

Cable-stayed bridges appeared only in the second half of the twentieth century, but their evolution has been extraordinarily rapid [14, 15]. Nowadays they are preferred for spans between 150m and almost 1000m [16]. The very large variety of shapes and the very large number of parameters open many possibilities and give the designer the opportunity of elegant designs. They offer great freedom of formal expression (Fig. 6) which many architects and engineers have tried to exploit [15].

![Fig. 6. The stunning Viaduc de Millau: an excellent example of a fruitful collaboration between architects and engineers.](image)

The basic resistant arrangement in cable-stayed bridges is formed by stays, deck and towers. Like in suspension bridges vertical loads move from their application points to the bottom of the stays thanks to the deck; then, both in harp shaped bridges (Fig. 7) and in fan shaped ones (Fig. 8), inside the stays, by a path in tension, they move from the deck to the tower. Vertical loads in order to enter the stays have to
deviate and to apply thrusts $H_i$ that find equilibrium with the thrusts applied by load deviating on the symmetrical bay. Differently from suspension bridges, in cable-stayed bridges the deck plays the basic role to receive these thrusts. While in harp shaped cable-stayed bridges since all the stays have the same inclinations, thrusts $H_i$ are all the same and so the axial stress increase in the deck due to these thrusts from the middle of the bay to the tower is linear, in fan shaped cable-stayed bridges this increase is not linear because the $H_i$ are different because of the different inclination of the stays. Load Path Method allows to graphically understand an important difference between suspension and cable-stayed bridges. While in cable-stayed bridges in a symmetrical vertical load condition, two half bays are in equilibrium because thrusts applied by vertical loads in their deviations can find equilibrium thanks to a path into the deck or directly in the pylon (Figs. 7, 8), in suspension bridges the overall equilibrium is possible only thanks to the presence of the anchor blocks; this means that while in cable-stayed bridges equilibrium is reached only by active forces (loads), in suspension bridges equilibrium can only be reached by passive forces (the weight of the anchor blocks plus the reaction of the soil) that balance the active ones.

![Fig. 8. Loads path in a fan shaped cable-stayed bridge.](image)

![Fig. 9. Loads path in the Alamillo Bridge (left) and a preliminary sketch by Santiago Calatrava (right [17]).](image)
By Load Path Method it is possible to immediately catch the behaviour of different types of bridges: the case in Fig. 9 of the loads path in the Alamillo bridge by Santiago Calatrava is a clear example. This is a cable-stayed bridge with only one bay. The particularity of this bridge is that thrusts generated in the deviation node of loads \( V_i \) coming from the deck are balanced by thrusts generated from the pylon self weight loads \( P_i \) that have to deviate in order to follow a path inside the inclined tower.

**SHAPE CONCEPTUAL DESIGN OF CABLE SUPPORTED BRIDGES BY LPM**

**Strain energy in suspension bridges**

According to LPM to optimise structural behaviour designer should search shapes that minimise strain energy \( D \).

In an orthogonal Cartesian system \( X\Omega Y \), with \( X \) axis horizontal, \( Y \) axis vertical, origin \( \Omega \) in the middle of the bay (Fig. 10) the following assumptions have been made:

- the bridge is geometrically symmetrical and symmetrically loaded only with gravity vertical loads;
- vertical loads are uniformly distributed, applied only on the deck and totally suspended;
- the part of the pylon under the deck is not considered in the analysis;
- there is an infinite distribution of vertical suspenders;
- deck and pylons are made with the same material;
- suspenders, main cable and pylon have constant transversal section.

![Fig. 10. Suspension bridge geometry.](image)

The following parameters are used in the analysis:

- \( r = \frac{h}{l} \) ratio of the pylon height (\( h \)), from the deck to the top, to the half span (\( l \)) of the deck (Fig. 10);
- \( \gamma = \frac{\varepsilon_s}{\varepsilon_p} \) ratio of the maximum design axial strain of the suspenders and of the main cable (\( \varepsilon_s \)) to the pylon maximum axial strain (\( \varepsilon_p \));
- \( D_{ref} = \frac{1}{2} q l^2 c_s \) is a reference strain energy; it represents the strain energy of a tie as long as half the deck, loaded with a tension axial load equal to the total vertical load applied on half a deck, and subjected to its maximum design strain \( \varepsilon_s \) (that is to say in the hypothesis to exploit its capacity).

Assuming equal to zero the strain energy of the anchor blocks (that is to say focusing the analysis on the central bay), the suspension bridge total strain energy \( D_{sb} \) is the summation of the strain energy of the main cable \( (D_c) \), of the suspenders \( (D_s) \) and of the pylon \( (D_p) \):

\[
D_{sb} = D_c + D_s + D_p \quad (3)
\]

The axial force in the main cable is

\[
T(x) = \frac{q x}{\sin \left[ a \tan \left( \frac{2 r x}{l} \right) \right]} \quad (4)
\]
The length of the infinitesimal part of the main cable is
\[
dL(x) = \sqrt{1 + \frac{4r^2}{l^2} x^2} \, dx
\]  
(5)

The axial strain in the main cable is
\[
\varepsilon(x) = \frac{x \sin[a \tan(2r)]}{l \sin[a \tan(\frac{2rx}{l})]} \varepsilon_s
\]  
(6)

And substituting in (1), the main cable strain energy is
\[
D_c = \frac{1}{2} q x^2 \frac{\sin[a \tan(2r)]}{2 l \sin[a \tan(\frac{2rx}{l})]} \varepsilon_s \sqrt{1 + \frac{4r^2}{l^2} x^2} \, dx
\]  
(7)

\[
\frac{D_c}{D_{ref}} = \frac{2r(5 + 28r^2 + 32r^4) + 3\sqrt{1 + 4r^2} \ln(2r + \sqrt{1 + 4r^2})}{32r^2(1 + 4r^2)}
\]  
(8)

The axial force in the elementary suspender is
\[
dN = q dx
\]  
(9)

The length of the suspender in the x position is
\[
L(x) = \frac{r}{l} x^2
\]  
(10)

And substituting in (1), the suspenders strain energy is
\[
D_s = \frac{1}{2} q \frac{r}{l} x^2 \varepsilon_s \, dx
\]  
(12)

\[
\frac{D_s}{D_{ref}} = \frac{r}{3}
\]  
(13)

The axial force in the pylon for the load of half a deck is
\[
N = q l
\]  
(14)

The pylon height is
\[
h = rl
\]  
(15)

And substituting in (1), the pylon strain energy becomes
\[
D_p = \frac{1}{2} q l^2 r^2 \varepsilon_p
\]  
(16)

\[
\frac{D_p}{D_{ref}} = \frac{r}{\gamma}
\]  
(17)

Finally, from (3) it is possible to obtain the total strain energy \(D_{sb}\) of the suspension bridge normalized by \(D_{ref}\).
\[ \frac{D_{cb}}{D_{ref}} = \frac{2r(5 + 28r^2 + 32r^4) + 3\sqrt{1 + 4r^2} \ln(2r + \sqrt{1 + 4r^2})}{32r^2(1 + 4r^2)} + \frac{r^3 + \frac{r}{3} + \frac{r}{y}}{y} \]  

(18)

**Strain energy in cable-stayed bridges**

In an orthogonal Cartesian system XΩY, with X axis horizontal, Y axis vertical, origin Ω at the intersection between the deck and the tower (Figs. 11, 12), the following assumptions have been made:

- the bridge is geometrically symmetrical and symmetrically loaded only with gravity vertical loads;
- vertical loads are uniformly distributed, applied only on the deck and totally suspended;
- the part of the pylon under the deck is not considered in the analysis;
- there is an infinite distribution of stays;
- deck and pylons are made with the same material;
- stays have constant transversal section;
- pylons have constant transversal section in the fan shaped bridge;
- pylons have not constant section in the harp shaped bridge in order to have in every section the same axial strain;
- only with reference to the actions generated by the analysed model, deck has in every section the same axial strain.

**Fig. 11.** Fan shaped cable-stayed bridge geometry.

**Fig. 12.** Harp shaped cable-stayed bridge geometry

In the analysis, the following parameters are used:

- \( r = \frac{h}{l} \) ratio of the pylon height (h), from the deck to the top, to the half span (l) of the deck (Figs. 11, 12);
- \( \gamma = \frac{\varepsilon_s}{\varepsilon_p} = \frac{\varepsilon_s}{\varepsilon_d} \) ratio of the stays maximum design axial strain (\( \varepsilon_s \)) to the maximum design axial strain of the pylon (\( \varepsilon_p \)) and of the deck (\( \varepsilon_d \));
- \( D_{ref} = \frac{1}{2}ql^2\varepsilon_s \) is a reference strain energy; it represents the strain energy of a tie as long as half the deck, loaded with a tension axial load equal to the total vertical load applied on half a deck, and subjected to its maximum design strain \( \varepsilon_s \) (that is to say in the hypothesis to exploit its capacity).

The total strain energy \( D_{cb} \) of the fan shaped cable-stayed bridge is the summation of the strain energy of
the stays \((D_s)\), of the deck \((D_d)\) and of the pylon \((D_p)\):

\[
D_{fcb} = D_s + D_d + D_p \tag{19}
\]

With the same procedure used for the suspension bridge it is possible to obtain

\[
\frac{D_{fcb}}{D_{ref}} = \frac{1}{3r} + \frac{1}{3r\gamma} + \frac{r}{\gamma} \tag{20}
\]

The total strain energy \(D_{hcb}\) of the harp shaped cable-stayed bridge is the summation of the strain energy of the stays \((D_s)\), of the deck \((D_d)\) and of the pylon \((D_p)\):

\[
D_{hcb} = D_s + D_d + D_p \tag{21}
\]

Using the same procedure of the suspension bridge it is possible to obtain

\[
\frac{D_{hcb}}{D_{ref}} = \frac{1}{2r} + \frac{1}{2r\gamma} + \frac{r}{2\gamma} \tag{22}
\]

Some considerations on the results

Equations (18), (20) and (22) can be easily represented graphically (Figs. 13, 14) to immediately evaluate, in terms of strain energy, which is the most “convenient solution”. In this paper the meaning of “convenient solution” is related to the design of cable supported bridge of a fixed geometry (span and pylons height) taking into account only the optimum structural behaviour for vertical loads; neglecting other actions and other aspects that can strongly influence the design choices (i.e. cost analysis, methods of construction, environmental restrictions, etc.) leads to assume that the satisfaction of the minimum value of strain energy coincides with the most “convenient solution”.

For example, in Figs. 13 and 14, the influence of the ratio \(r=h/l\) (that represents the geometry of the bridges)
on the ratio $D/D_{ref}$ is shown in the case of $\gamma=1$ and $\gamma=4$.

In the first case ($\gamma=1$; Fig.13), if $r<0.85$ suspension bridges dissipate less strain energy than the cable-stayed ones with a minimum value for $r=0.55$; if $r>0.85$ the harp shaped cable-stayed bridges dissipate less strain energy than the other two kinds of bridges.

In the second case ($\gamma=4$; Fig.14), if $r<0.42$ fan shaped cable-stayed bridges dissipate less strain energy than the other two types of bridges; if $r>0.70$ the harp shaped cable-stayed bridges dissipate less strain energy than the other two bridges.

![Graph](image.png)

Fig. 14. $D/D_{ref}$ in the case of $\gamma=4$.

**CONCLUSIONS**

Load path method is an instrument to analyse structural continuum. It seems to have the peculiar capacity to easily catch the physical behaviour of a structure, from its global behaviour to the most accurate details. In this paper a first proposal of the use of LPM basic principles to shape conceptual design of bridges has been presented. In geometrical contours of bridges, maybe more than in other architectural works, it is possible to recognise the expression, almost exclusive, of structural functionality.

This first approach presented has been made in order to show LPM versatility and effectiveness. In fact the model is capable to simply simulate structural behaviour by the perception of the adhesion of its geometrical profiles to the form. In this way, in architecture, it is possible to identify the measure in which structural shapes are crossed by thrusts paths rather than loads paths, and vice versa, as an element characterizing structural shape.

Besides, some examples of numerical applications of LPM to the optimum structural behaviour of cable supported bridge with vertical loads have been presented. The aim has been to show that LPM seems also to successfully conciliate the necessity to get a numerical solution and to never loose touch with the perception of the synthesis of physical structural behaviour.

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